Safe Functional Reactive Programming through Dependent Types

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Reactive Programming

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- Examples: MP3 players, robot controllers, video games, aeroplane control systems...
- Contrast with transformational programs, which take all input at the start of execution and produce all output at the end (e.g. a compiler).

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- Functional Reactive Programming (FRP) differs in that it is very expressive, but lacking in these guarantees.
- This work is about using dependent types to get some of these safety guarantees within FRP (without sacrificing expressiveness).

Outline

1. Motivation
2. Outline
3. Dependent Types in FRP
4. Functional Reactive Programming (FRP)
5. New Type System
6. Safe Feedback Loops
7. Uninitialised Signals
8. Summary

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- The implementation serves as a proof of the soundness of the type system. (Agda checks totality and termination.)

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Example: Robot Controller

$$\text{RobotController} = \text{SF Sensor ControlValue}$$

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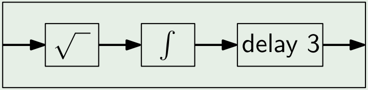
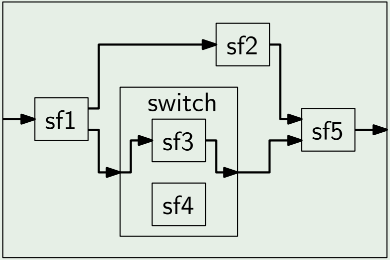
  - Decoupled signal functions: current output only depends upon past inputs (e.g. time delay).
- We compose signal functions to form signal function networks.

Example: Composing Signal Functions



Synchronous Data-Flow Networks

Example: A Signal Function Network



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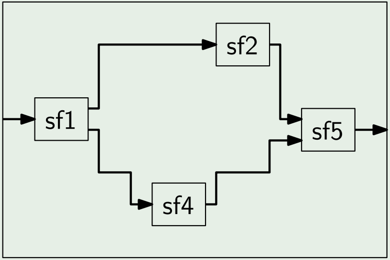

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- To address this, we introduce signal vectors: conceptually heterogeneous vectors of signals that allows us to precisely identify signals (and their time domains) in the types.

Signal Descriptors

Descriptor Definitions

data SigDesc : Set **where**
  E : Set → SigDesc
  C : Set → SigDesc

SVDesc : Set
SVDesc = List SigDesc

Example: A Signal Vector Descriptor

svdExample : SVDesc
svdExample = $(C\ \mathbb{R} :: E\ \mathrm{Bool} :: C\ \mathbb{Z} :: [\,])$

Signal Functions

Original SF Type

$$SF : \mathsf{Set} \longrightarrow \mathsf{Set} \longrightarrow \mathsf{Set}$$

Revised SF Type

$$SF : \mathsf{SVDesc} \longrightarrow \mathsf{SVDesc} \longrightarrow \mathsf{Set}$$

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Revised SF Type

$$\mathsf{SF} : \mathsf{SVDesc} \longrightarrow \mathsf{SVDesc} \longrightarrow \mathsf{Set}$$

Example: Some Primitive Signal Functions

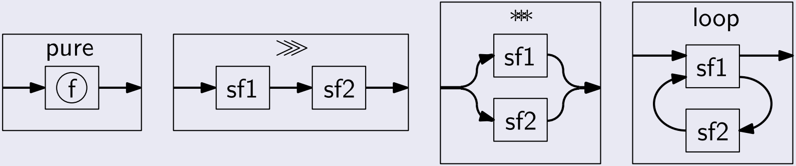
$$\begin{aligned}
\mathsf{now} &: \mathsf{SF}\ []\ [\mathsf{E}\ \mathsf{Unit}] \\
\mathsf{time} &: \mathsf{SF}\ []\ [\mathsf{C}\ \mathsf{Time}] \\
\mathsf{edge} &: \mathsf{SF}\ [\mathsf{C}\ \mathsf{Bool}]\ [\mathsf{E}\ \mathsf{Unit}] \\
\int &: \mathsf{SF}\ [\mathsf{C}\ \mathbb{R}]\ [\mathsf{C}\ \mathbb{R}]
\end{aligned}$$

Constructing Signal Functions

Primitive Combinators

$$
\begin{aligned}
\mathsf{pure} &: (a \rightarrow b) \rightarrow SF\ [C\ a]\ [C\ b] \\
\_\ggg\_ &: SF\ as\ bs \rightarrow SF\ bs\ cs \rightarrow SF\ as\ cs \\
\_{*\!*\!*}\_ &: SF\ as\ cs \rightarrow SF\ bs\ ds \rightarrow SF\ (as +\!\!+ bs)\ (cs +\!\!+ ds) \\
\mathsf{loop} &: SF\ (as +\!\!+ cs)\ (bs +\!\!+ ds) \rightarrow SF\ ds\ cs \rightarrow SF\ as\ bs
\end{aligned}
$$

Graphical Representations

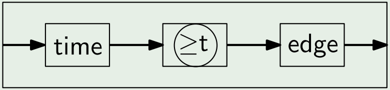


Constructing Signal Functions

Example: The *after* Signal Function

The signal function after t produces an event at time t.

after : Time → SF [] [E Unit]
after t = time ⋙ pure (_⩽_ t) ⋙ edge



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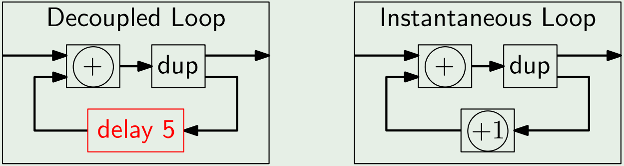
- Feedback loops are well defined if somewhere in the cycle they are broken by a decoupled signal function.
- Reminder: a signal function is decoupled if its current output only depends upon its past inputs.
- Methods of decoupling: time delays, constants, some primitives (e.g. integration using the rectangle rule). . .

Examples: Loops



Existing Approaches to Decoupling

Relying on the programmer to correctly define loops.

- Does not restrict expressiveness.
- Easy to introduce bugs into programs.
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Explicit use of a decoupling primitive in all recursive definitions.

- Can be confirmed as safe by the type checker (conservatively).
- Limits expressiveness (cannot use dynamic or higher order signal functions for decoupling).
- Most synchronous data-flow languages take this approach.

Our Approach: Decoupledness in the Types

We index signal function types with a boolean to denote their decoupledness.

Primitive Combinators Indexed by Decoupledness

$$\begin{array}{lll}
\mathrm{pure} & : & (\mathrm{a} \rightarrow \mathrm{b}) \rightarrow \mathrm{SF}\ [\mathrm{C\ a}]\ [\mathrm{C\ b}]\ \mathrm{false} \\
\_\ggg\_ & : & \mathrm{SF\ as\ bs}\ d_1 \rightarrow \mathrm{SF\ bs\ cs}\ d_2 \rightarrow \mathrm{SF\ as\ cs}\ (d_1 \vee d_2) \\
\_{*}{*}{*}\_ & : & \mathrm{SF\ as\ cs}\ d_1 \rightarrow \mathrm{SF\ bs\ ds}\ d_2 \rightarrow \mathrm{SF}\ (\mathrm{as} +\!\!+ \mathrm{bs})\ (\mathrm{cs} +\!\!+ \mathrm{ds})\ (d_1 \wedge d_2) \\
\mathrm{loop} & : & \mathrm{SF}\ (\mathrm{as} +\!\!+ \mathrm{cs})\ (\mathrm{bs} +\!\!+ \mathrm{ds})\ d \rightarrow \mathrm{SF\ ds\ cs\ true} \rightarrow \mathrm{SF\ as\ bs}\ d
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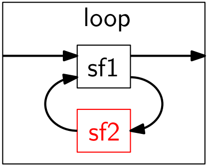

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$\_ \ggg \_ : \text{SF as bs}\ d_1 \to \text{SF bs cs}\ d_2 \to \text{SF as cs}\ (d_1 \vee d_2)$

$\_ \ast\!\!\ast\!\!\ast \_ : \text{SF as cs}\ d_1 \to \text{SF bs ds}\ d_2 \to \text{SF}\ (\text{as} +\!\!+ \text{bs})\ (\text{cs} +\!\!+ \text{ds})\ (d_1 \wedge d_2)$

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Examples: Primitive Signal Functions Indexed by Decoupledness

$\text{now} : \text{SF}\ []\ [\text{E Unit}]\ \text{true}$

$\text{time} : \text{SF}\ []\ [\text{C Time}]\ \text{true}$

$\text{edge} : \text{SF}\ [\text{C Bool}]\ [\text{E Unit}]\ \text{false}$

$\int : \text{SF}\ [\text{C}\ \mathbb{R}]\ [\text{C}\ \mathbb{R}]\ ?$

Uninitialised Signals

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- The decoupled signal function pre introduces an infinitesimal time delay in a continuous-time signal.
- But this also means the signal is initially undefined.

Initialisation Primitives

pre : SF [C a] [C a] true

initialise : a → SF [C a] [C a] false

iPre : a → SF [C a] [C a] true

Uninitialised Signals

Boolean Synonyms

$$\begin{aligned}\mathsf{Init} &= \mathsf{Bool}\\ \mathsf{init} &= \mathsf{true}\\ \mathsf{unin} &= \mathsf{false}\end{aligned}$$

Adding Initialisation to Signal Descriptors

```
data SigDesc : Set where
  E :          Set → SigDesc
  C : Init  →  Set → SigDesc
```

Note that event signals are only defined at discrete points in time, so there is no need to initialise them.

Uninitialised Signals

Primitives updated with Initialisation Descriptors

$$\begin{aligned}
&\mathsf{pure} &&: (a \rightarrow b) \rightarrow SF\ [C\ i\ a]\ [C\ i\ b]\ \mathsf{false}\\
&\mathsf{pre} &&: SF\ [C\ \mathsf{init}\ a]\ [C\ \mathsf{unin}\ a]\ \mathsf{true}\\
&\mathsf{initialise} &&: a \rightarrow SF\ [C\ \mathsf{unin}\ a]\ [C\ \mathsf{init}\ a]\ \mathsf{false}\\
&\mathsf{iPre} &&: a \rightarrow SF\ [C\ \mathsf{init}\ a]\ [C\ \mathsf{init}\ a]\ \mathsf{true}
\end{aligned}$$

Summary

- FRP and synchronous data-flow languages make a trade-off between expressiveness and safety.
- Dependent types allow us to have FRP with safety guarantees, while retaining dynamic higher-order data-flow.
- One such safety guarantee is the absence of instantaneous feedback loops.
- Another is that all signals (that require it) are correctly initialised.
- See our paper for further details: `http://www.cs.nott.ac.uk/~nas/icfp09.pdf`