Safe Functional Reactive Programming through Dependent Types

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Reactive Programming

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- Examples: MP3 players, robot controllers, video games, aeroplane control systems...
- Contrast with transformational programs, which take all input at the start of execution and produce all output at the end (e.g. a compiler).

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- Functional Reactive Programming (FRP):
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  - Lacks many safety guarantees.
- This work: using dependent types to get safety guarantees within FRP without sacrificing expressiveness.

Outline

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- In development: a Haskell implementation (using GHC language extensions).

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- Signal functions are (conceptually) functions mapping signals to signals.

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Example: Robot Controller

$$\text{RobotController} = \text{SF Sensor ControlValue}$$

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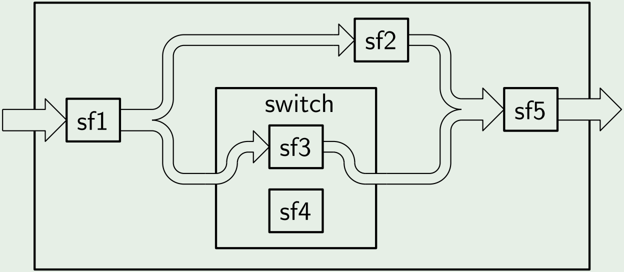
Implementing Signal Functions

- In practise, FRP implementations run signal functions over a discrete sequence of time samples (synchronously).
- This is hidden by the signal function abstraction.

Synchronous Data-Flow Networks

Example: A Signal Function Network



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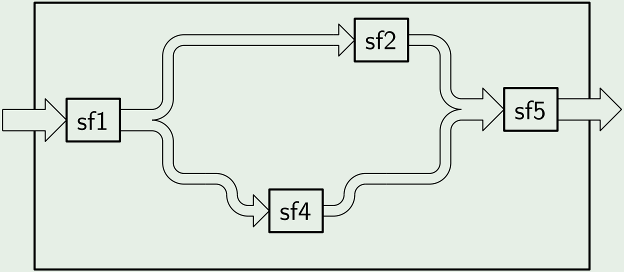

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- Problems:
  - Insufficiently abstract to exploit their discrete properties.
  - Can lead to conceptual errors by the programmer.

Signal Vectors

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Example: Some Primitive Signal Functions

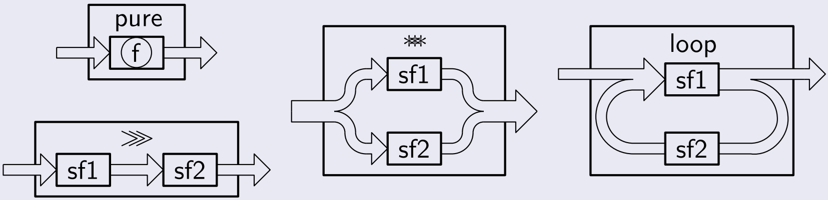
$$\begin{aligned}
\mathsf{now} &: \mathsf{SF}\ [\,]\ [\mathsf{E}\ \mathsf{Unit}] \\
\mathsf{time} &: \mathsf{SF}\ [\,]\ [\mathsf{C}\ \mathsf{Time}] \\
\mathsf{edge} &: \mathsf{SF}\ [\mathsf{C}\ \mathsf{Bool}]\ [\mathsf{E}\ \mathsf{Unit}] \\
\int &: \mathsf{SF}\ [\mathsf{C}\ \mathbb{R}]\ [\mathsf{C}\ \mathbb{R}]
\end{aligned}$$

Constructing Signal Functions

Primitive Combinators

$$
\begin{aligned}
\mathsf{pure} &: (a \to b) \to \mathsf{SF}\ [\mathsf{C}\ a]\ [\mathsf{C}\ b] \\
\_ \ggg \_ &: \mathsf{SF}\ as\ bs \to \mathsf{SF}\ bs\ cs \to \mathsf{SF}\ as\ cs \\
\_ \ast\!\!\ast\!\!\ast \_ &: \mathsf{SF}\ as\ cs \to \mathsf{SF}\ bs\ ds \to \mathsf{SF}\ (as +\!\!+\ bs)\ (cs +\!\!+\ ds) \\
\mathsf{loop} &: \mathsf{SF}\ (as +\!\!+\ cs)\ (bs +\!\!+\ ds) \to \mathsf{SF}\ ds\ cs \to \mathsf{SF}\ as\ bs
\end{aligned}
$$

Graphical Representations

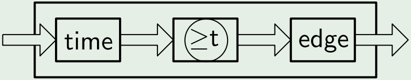


Constructing Signal Functions

Example: The *after* Signal Function

The signal function after t produces an event at time t.

$$\text{after} : \text{Time} \rightarrow \text{SF}\ [\,]\ [\text{E Unit}]$$
$$\text{after t} = \text{time} \ggg \text{pure}\ (\geqslant \text{t}) \ggg \text{edge}$$



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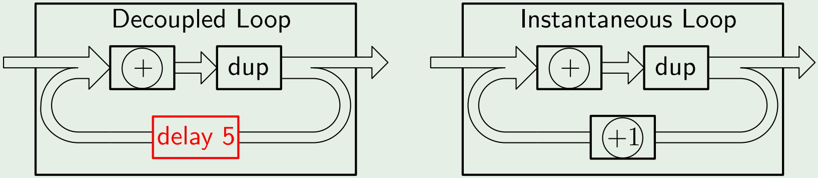
- Feedback loops are well defined if somewhere in the cycle they are broken by a decoupled signal function.
- Decoupled signal function: current output only depends upon its past inputs.
- Methods of decoupling: time delays, infinitesimal delays, some primitives (e.g. integration using the rectangle rule)...

Examples: Loops



Existing Approaches to Decoupling

Relying on the programmer to correctly define loops.

- Does not restrict expressiveness.
- Easy to introduce bugs into programs.
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Explicit use of a decoupling primitive in all recursive definitions.

- Can be confirmed as safe by the type checker (conservatively).
- Limits expressiveness (in particular, structural dynamism and higher-order signal functions).
- Most synchronous data-flow languages take this approach.

Our Approach: Decoupledness in the Types

Index signal functions by booleans to denote decoupledness.

Primitive Combinators Indexed by Decoupledness

$$
\begin{array}{ll}
\mathrm{pure} & : (a \to b) \to \mathrm{SF}\ [\mathrm{C}\ a]\ [\mathrm{C}\ b]\ \textcolor{red}{\mathrm{false}} \\
\_ \ggg \_ & : \mathrm{SF}\ as\ bs\ \textcolor{red}{d_1} \to \mathrm{SF}\ bs\ cs\ \textcolor{red}{d_2} \to \mathrm{SF}\ as\ cs\ \textcolor{red}{(d_1 \vee d_2)} \\
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\mathrm{loop} & : \mathrm{SF}\ (as +\!\!+ cs)\ (bs +\!\!+ ds)\ \textcolor{red}{d} \to \mathrm{SF}\ ds\ cs\ \textcolor{red}{\mathrm{true}} \to \mathrm{SF}\ as\ bs\ \textcolor{red}{d}
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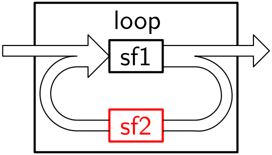

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Initialisation Primitives

$$\begin{array}{ll} \text{pre} & : \text{SF [C a] [C a] true} \\ \text{initialise} & : \text{a} \rightarrow \text{SF [C a] [C a] false} \\ \text{iPre} & : \text{a} \rightarrow \text{SF [C a] [C a] true} \end{array}$$

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Primitives updated with Initialisation Descriptors

$$
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\text{pre} & : \text{SF } [\text{C init a}]\ [\text{C unin a}] \text{ true} \\
\text{initialise} & : a \rightarrow \text{SF } [\text{C unin a}]\ [\text{C init a}] \text{ false} \\
\text{iPre} & : a \rightarrow \text{SF } [\text{C init a}]\ [\text{C init a}] \text{ true}
\end{array}
$$

Boolean Synonyms

$$
\begin{array}{lcl}
\text{init} & = & \text{true} \\
\text{unin} & = & \text{false}
\end{array}
$$

Event signals are only defined at discrete points in time, so there is no need to ensure initialisation.

Summary

- FRP and synchronous data-flow languages make a trade-off between expressiveness and safety.
- Dependent types allow us to have FRP with safety guarantees, while retaining dynamic higher-order data-flow.
- Examples:
  - Absence of instantaneous feedback loops.
  - Correct initialisation of signals.
- See the paper for further details.