The Constrained-Monad Problem

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Monads in Haskell

```
{-# LANGUAGE KindSignatures #-}
```

The Monad Type Class

```
class Monad (m :: ∗ → ∗) where
  return :: a → m a
  ( ≫= ) :: m a → (a → m b) → m b
```

The Monad Laws

- return a ≫= g ≡ g a (left-identity law)
- m ≫= return ≡ m (right-identity law)
- (m ≫= g) ≫= h ≡ m ≫= (λ x → g x ≫= h) (associativity law)

Sets in Haskell

```
import Data.Set
```

Selected functions from the Data.Set library

```
singleton :: a → Set a
toList    :: Set a → [a]
fromList  :: Ord a ⇒ [a] → Set a
unions    :: Ord a ⇒ [Set a] → Set a
```

Sets in Haskell

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Monadic Set Operations

```
returnSet :: a → Set a
returnSet = singleton

bindSet :: Ord b ⇒ Set a → (a → Set b) → Set b
bindSet s k = unions (map k (toList s))
```

Sets in Haskell

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Monadic Set Operations

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returnSet :: a → Set a
returnSet = singleton

bindSet :: Ord b ⇒ Set a → (a → Set b) → Set b
bindSet s k = unions (map k (toList s))

instance Monad Set where
  return  = returnSet
  ( ≫= ) = bindSet        -- does not type check
```

Vectors

A Vector Representation

type Vec (a :: $*$) = (a → Double)

Vectors

A Vector Representation

```
type Vec (a :: ∗) = (a → Double)
```

Monadic Vector Operations

```
class Finite (a :: ∗) where
  enumerate :: [a]

returnVec :: Eq a ⇒ a → Vec a
returnVec a = λ b → if a == b then 1 else 0

bindVec :: Finite a ⇒ Vec a → (a → Vec b) → Vec b
bindVec v k = λ b → sum [v a × (k a) b | a ← enumerate]
```

Vectors

A Vector Representation

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bindVec :: Finite a ⇒ Vec a → (a → Vec b) → Vec b
bindVec v k = λ b → sum [v a × (k a) b | a ← enumerate]

instance Monad Vector where
   return  = returnVec       -- does not type check
   ( ≫= ) = bindVec         -- does not type check
```

Embedded Domain Specific Languages

{-# LANGUAGE GADTs #-}

Embedding Monadic Operations

```
data EDSL :: * → * where
  IfThenElse :: EDSL Bool → EDSL a → EDSL a → EDSL a
  Return     :: a                             → EDSL a
  Bind       :: EDSL x → (x → EDSL a)         → EDSL a
  ...
```

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instance Monad EDSL where
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   ...

instance Monad EDSL where
   return = Return
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compile :: Reifiable a ⇒ EDSL a → Code
```

Embedded Domain Specific Languages

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```

Embedding Monadic Operations

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data EDSL :: ∗ → ∗ where
  IfThenElse :: EDSL Bool → EDSL a → EDSL a         → EDSL a
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  . . .

instance Monad EDSL where
  return  = Return
  ( ≫= ) = Bind      -- does not type check

compile :: Reifiable a ⇒ EDSL a → Code
```

Why is this a Problem?

- A Monad instance is useful because the Haskell language and libraries provide a significant amount of infrastructure to support arbitrary monads.
- The problem generalises from monads to any type class with polymorphic class methods.
- This talk will mostly be about monads, but will conclude with some other examples.
- Our solution generalises to some, but not all, type classes.
- Future work:
  - characterising the type classes for which it works;
  - extending/adapting the solution to other type classes.

Constraint Kinds

```
{-# LANGUAGE ConstraintKinds #-}

import GHC.Exts (Constraint)
```

Constraint Kinds in GHC

The kind of a fully applied type class is the literal kind Constraint.
For example:

```
Ord   :: * → Constraint
Monad :: (* → *) → Constraint
```

A Partial Solution: Restricted Type Classes

```
{-# LANGUAGE MultiParamTypeClasses, InstanceSigs #-}
```

Restricted Monad Class

```
class RMonad (c :: ∗ → Constraint) (m :: ∗ → ∗) where
  return :: c a          ⇒ a → m a
  ( ≫= ) :: (c a, c b) ⇒ m a → (a → m b) → m b
```

A Partial Solution: Restricted Type Classes

```
{-# LANGUAGE MultiParamTypeClasses, InstanceSigs #-}
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Restricted Monad Class

```
class RMonad (c :: ∗ → Constraint) (m :: ∗ → ∗) where
   return :: c a          ⇒ a → m a
   ( ≫= ) :: (c a, c b) ⇒ m a → (a → m b) → m b
```

Example: Set and Ord

```
instance RMonad Ord Set where
   return  :: Ord a ⇒ a → Set a
   return  = returnSet
   ( ≫= ) :: (Ord a, Ord b) ⇒ Set a → (a → Set b) → Set b
   ( ≫= ) = bindSet
```

An alternative: Embedding and Normalisation

- An alternative is to embed the type in another data type that does form a monad.

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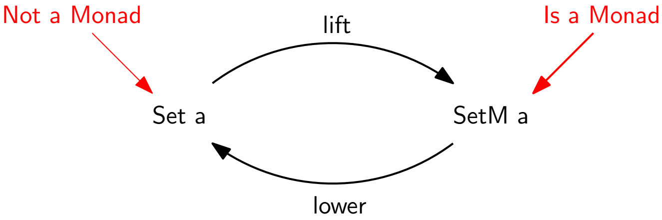


Not a Monad

lift

Is a Monad

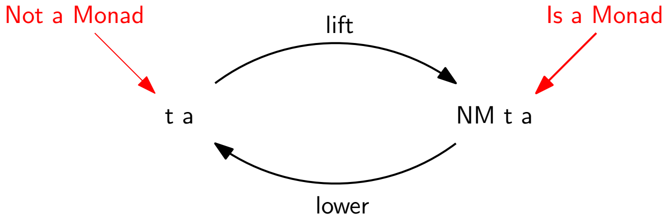
Set a

SetM a

lower

An alternative: Embedding and Normalisation

- An alternative is to embed the type in another data type that does form a monad.



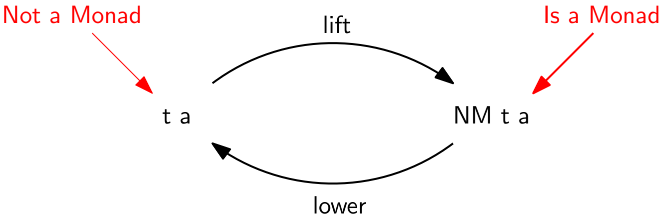
Not a Monad

Is a Monad

$$\begin{tikzcd}
t\ a \arrow[r, bend left, "lift"] & NM\ t\ a \arrow[l, bend left, "lower"]
\end{tikzcd}$$

An alternative: Embedding and Normalisation

- An alternative is to embed the type in another data type that does form a monad.



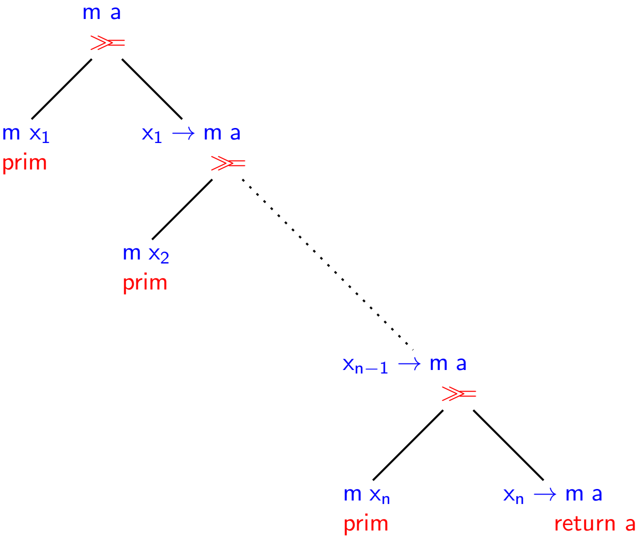
Not a Monad

Is a Monad

$$\begin{tikzcd} \text{t a} \arrow[r, bend left, "\text{lift}"] & \text{NM t a} \arrow[l, bend left, "\text{lower}"] \end{tikzcd}$$

- The key ideas are:
  - NM represents a monadic computation in a normal form;
  - the lift and lower functions enforce the constraint.

A Normal Form for Monadic Computations



$m\ a$

$\gg=$

$m\ x_1$

prim

$x_1 \to m\ a$

$\gg=$

$m\ x_2$

prim

$x_{n-1} \to m\ a$

$\gg=$

$m\ x_n$

prim

$x_n \to m\ a$

return a

Embedding Constrained Monadic Computations

```
{-# LANGUAGE GADTs #-}
```

Normalised Monads as a GADT

```
data NM :: (∗ → ∗) → ∗ → ∗ where
  Return :: a                       → NM t a
  Bind   :: t x → (x → NM t a) → NM t a
```

Embedding Constrained Monadic Computations

{-# LANGUAGE GADTs #-}

Constrained Normalised Monads as a GADT

```
data NM :: (* → Constraint) → (* → *) → * → * where
   Return :: a                                → NM c t a
   Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Embedding Constrained Monadic Computations

```
{-# LANGUAGE GADTs #-}
```

Constrained Normalised Monads as a GADT

```
data NM :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
  Return :: a                                → NM c t a
  Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Constrained Normalised Monads are (standard) Monads!

```
instance Monad (NM c t) where
  return :: a → NM c t a
  return = Return

  ( ≫= ) :: NM c t a → (a → NM c t b) → NM c t b
  (Return a)  ≫= k = k a                          -- left-identity law
  (Bind tx h) ≫= k = Bind tx (λ x → h x ≫= k)     -- associativity law
```

Embedding Constrained Monadic Computations

```
{-# LANGUAGE GADTs #-}
```

Constrained Normalised Monads as a GADT

```
data NM :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
   Return :: a                                  → NM c t a
   Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Lifting Primitive Operations

```
lift :: c a ⇒ t a → NM c t a
lift ta = Bind ta Return       -- right-identity law
```

Embedding Constrained Monadic Computations

```
{-# LANGUAGE GADTs, RankNTypes, ScopedTypeVariables #-}
```

Constrained Normalised Monads as a GADT

```
data NM :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
  Return :: a                                → NM c t a
  Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Lowering Monadic Computations

```
lower :: ∀ a c t. (a → t a) → (∀ x. c x ⇒ t x → (x → t a) → t a) → NM c t a → t a
lower ret bind = lower′
  where
    lower′ :: NM c t a → t a
    lower′ (Return a)  = ret a
    lower′ (Bind tx k) = bind tx (lower′ ∘ k)
```

Embedding Constrained Monadic Computations

```
{-# LANGUAGE GADTs, RankNTypes, ScopedTypeVariables #-}
```

Constrained Normalised Monads as a GADT

```
data NM :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
   Return :: a                                    → NM c t a
   Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Example: Set and Ord

```
type SetM a = NM Ord Set a

liftSet :: Ord a ⇒ Set a → SetM a
liftSet = lift

lowerSet :: Ord a ⇒ SetM a → Set a
lowerSet = lower returnSet bindSet
```

Embedding Constrained Monadic Computations

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Constrained Normalised Monads as a GADT

```
data NM :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
  Return :: a                                 → NM c t a
  Bind   :: c x ⇒ t x → (x → NM c t a) → NM c t a
```

Folding Monadic Computations

```
fold :: ∀ a c r t. (a → r) → (∀ x. c x ⇒ t x → (x → r) → r) → NM c t a → r
fold ret bind = fold′
  where
    fold′ :: NM c t a → r
    fold′ (Return a)  = ret a
    fold′ (Bind tx k) = bind tx (fold′ ∘ k)
```

Embedding Constrained Functor Computations

Constrained Normalised Functors as a GADT

```
data NF :: (* → Constraint) → (* → *) → * → * where
   FMap :: c x ⇒ (x → a) → t x → NF c t a
```

Embedding Constrained Functor Computations

Constrained Normalised Functors as a GADT

```
data NF :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
   FMap :: c x ⇒ (x → a) → t x → NF c t a
```

Constrained Normalised Functors are (standard) Functors

```
instance Functor (NF c t) where
   fmap :: (a → b) → NF c t a → NF c t b
   fmap g (FMap h tx) = FMap (g ∘ h) tx        -- composition law
```

Embedding Constrained Functor Computations

Constrained Normalised Functors as a GADT

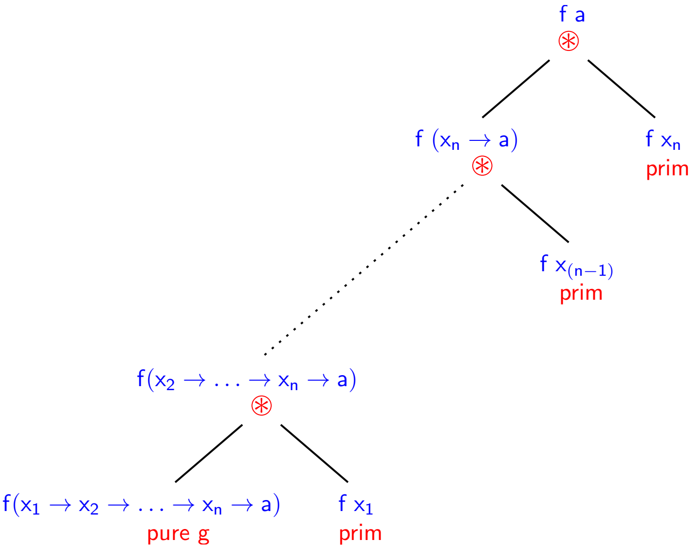
```
data NF :: (∗ → Constraint) → (∗ → ∗) → ∗ → ∗ where
  FMap :: c x ⇒ (x → a) → t x → NF c t a
```

Lifting and Lowering

```
liftNF :: c a ⇒ t a → NF c t a
liftNF ta = FMap id ta          -- identity law

lowerNF :: (∀ x. c x ⇒ (x → a) → t x → t a) → NF c t a → t a
lowerNF fmp (FMap g tx) = fmp g tx
```

A Normal Form for Applicative Computations



$f\ a$

$\circledast$

$f\ (x_n \to a)$

$\circledast$

$f\ x_n$

prim

$f\ x_{(n-1)}$

prim

$f (x_2 \to \ldots \to x_n \to a)$

$\circledast$

$f (x_1 \to x_2 \to \ldots \to x_n \to a)$

pure g

$f\ x_1$

prim

Remarks

- The normalisation solution requires a normal form where all existential types are parameters on primitive operations. E.g.
    - this is true of Category
    - but not Arrow

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- The monadic normalisation is the same as used by Unimo [Lin06], MonadPrompt [IF08], and Operational [Apf10], and brings the same benefits:
    - enforces the monad laws
    - separates structure from interpretation
    - allows multiple interpretations

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- The first use of normalisation to overcome the constrained-monad problem was by the RMonad library [SG08].

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- An alternative means of normalising is to use a continuation transformer [PAS12].

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    - separates structure from interpretation
    - allows multiple interpretations
- The first use of normalisation to overcome the constrained-monad problem was by the RMonad library [SG08].
- An alternative means of normalising is to use a continuation transformer [PAS12].
- Normalisation preserves semantics, but can change the operational behaviour of the monad.

Further Reading

See our paper for more details:

Neil Sculthorpe, Jan Bracker, George Giorgidze and Andy Gill.
The Constrained-Monad Problem.
In *International Conference on Functional Programming*. ACM, 2013.

`http://www.ittc.ku.edu/~neil/publications.html`.

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